

BUDHA DAL PUBLIC SCHOOL PATIALA
FIRST TERM EXAMINATION (11 September 2023)

Class - XII

Paper- Applied Mathematics

Time: 3hrs.

M.M. 80

General Instructions:

1. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer type questions of 2 marks each.
4. Section C has 6 Short Answer type questions of 3 marks each.
5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 case based studies of 4 marks each.

Section - A

1. $(62 + 53) \bmod 7$ is equal to

- a) 3 b) 5 c) 7 d) 9

2. Time series analysis helps

- a) to make predictions
- b) to compare two or more series
- c) to know behaviour of business
- d) all of the above

3. Solution set of inequalities $6 \leq -3(2x - 4) < 12$ is

- a) $(-\infty, 1]$ b) $(0, 1]$ c) $(0, 1] \cup [1, \infty)$ d) $[1, \infty)$

4. If $A^2 + A + I = 0$, then A^{-1} is equal to

- a) $A - I$ b) $I - A$ c) $-(I + A)$ d) none

5. If $A = \begin{bmatrix} 2 & x+7 \\ 2x-3 & x+8 \end{bmatrix}$ is symmetric, then x is equal to

- a) 1 b) 0 c) 5 d) 4

6. A factory produces bulbs. The probability that any bulb is defective is $1/50$ and they are packed in 10 boxes. From a single box, find the probability that none of the bulb is defective.

- a) $\left(\frac{1}{50}\right)^{10}$ b) $\left(\frac{49}{50}\right)^{10}$ c) $\left(\frac{1}{50}\right)^9$ d) $\left(\frac{49}{50}\right)^9$

7. The best fitting trend is one in which the sum of squares of residual is

- a) maximum b) minimum c) zero d) none of these

8. In what ratio must a shopkeeper mix two types of oranges worth Rs. 60/kg and Rs. 90/kg respectively so as to get a mixture at Rs. 80/kg?
- a) 1:1 b) 1:2 c) 2:1 d) 1:3
9. If $A = \begin{bmatrix} -3 & x \\ y & 5 \end{bmatrix}$ and $A = A'$, then
- a) $x = 5, y = -3$ b) $x = -3, y = 5$ c) $x = y$ d) all of above
10. If A is a square matrix, of order 3 and $|A| = 5$, then value of $|2A'|$ is
- a) -10 b) 10 c) -40 d) 40
11. If $x = at^2, y = 2at$, then $\frac{dy}{dx}$ is
- a) t b) $\frac{1}{t}$ c) t^2 d) $\frac{1}{t^2}$
12. If $C(x)$ and $R(x)$ are respectively Cost function and Revenue function, then Profit function $P(x)$ is given by
- a) $P(x) = R(x)$ b) $P(x) = C(x) + R(x)$ c) $P(x) = R(x) - C(x)$ d) $P(x) = R(x) \cdot C(x)$
13. The interval $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is decreasing in
- a) $(\infty, -1] \cup [0, 2]$ b) $[-1, 0] \cup [2, \infty)$ c) both d) none
14. If $|x - 2| \geq 7, x \in R$, then
- a) $x \in [-5, 9]$ b) $x \in (-5, 9]$ c) $x \in (-\infty, -5] \cup [9, \infty)$ d) $x \in (-\infty, -5] \cup (9, \infty)$
15. Given that x, y and b are real numbers and $x < y, b < 0$, then
- a) $\frac{x}{b} < \frac{y}{b}$ b) $\frac{x}{b} \leq \frac{y}{b}$ c) $\frac{x}{b} > \frac{y}{b}$ d) $\frac{x}{b} \geq \frac{y}{b}$
16. The normal distribution curve is symmetrical about
- a) $X = \mu$ b) $X = \sigma$ c) $X = \frac{\mu}{\sigma}$ d) $X = \frac{\sigma}{\mu}$
17. For a random variable, $E(X) = 3$ and $E(X^2) = 11$ then variance of X is
- a) 8 b) 5 c) 7 d) 2

18. If the total cost of producing x units of a commodity is given by

$$C(x) = \frac{1}{3}x^3 + x^2 - 15x + 3000, \text{ then the marginal cost when } x = 5 \text{ is}$$

- a) Rs. 25 b) Rs. 20 c) Rs. 30 d) Rs. 50

Assertion - Reason Based Questions

The following questions consists of two statements - Assertion (A) and Reason (R). Answer the question selecting appropriate option given below:

- a) Both A and R are true and R is correct explanation for R.
 b) Both A and R are true but R is not correct explanation for R.
 c) A is true but R is false.
 d) A is false but R is true.
19. Let A and B two square matrices of order 2

Statement 1 : $A(\text{adj } A) = |A|I_2$

Statement 2 : $\text{adj } (AB) = (\text{adj } A)(\text{adj } B)$

20. Statement 1 : If a random variable X has following probability distribution

X	0	1	2	3
$P(X)$	K^2	$2K^2$	K	K

then value of $K = \frac{1}{3}$

Statement 2 : If a random variable X assumes value x_1, x_2, \dots, x_n then

$$P(X = x_1) + P(X = x_2) + \dots + P(X = x_n) = 1$$

Section - B

21. Solve the following inequality

$$2y - 3 < y + 2 \leq 3y + 5$$

22. Construct a 2×3 matrix whose elements a_{ij} are given by $a_{ij} = (i + 2j)^2$

23. Solve following system of equation by Cramer's Rule : $2x - y = 17, 3x + 5y = 6$

24. Find the rate of change of the area of a circle with respect to its radius r where $r = 10\text{cm}$.

25. Suppose 2% of the items made by a factory are defective. Find the probability that there are defective items in a sample of 100 items selected at random. (Given $e^{-2} = 0.135$)

Section - C

26. A cistern can be filled by an inlet pipe in 20 hours and can be emptied by an outlet pipe in 25 hours. Both the pipes are opened, after 10 hours, the outlet pipe is closed, find the total time taken to fill the cistern.

27. Find the inverse of matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and hence show that $A^{-1}A = I$

28. Five cards are drawn successively with replacement from a well-shuffled pack of 52 cards. What is the probability that

- a) All 5 cards are spades
- b) Only 3 cards are spades
- c) None is spade?

29. The average number, in lakhs, of working days lost in strikes during each year of the period (1981-90) was as under:

1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

Calculate three yearly moving average for above information.

30. A wire of length 50m is to be cut into two pieces. One piece is bent in the shape of a square and other in the shape of a circle. What should be the lengths of each piece so that the combined area of the two shapes is minimum?

31. In a game, A can give 20 points to B, A can give 32 points to C and B can give 15 points to C. How many points make the game?

Section - D

32. Fit a straight line trend by the method of least squares and tabulate the trend values from the data

Year	2004	2005	2006	2007	2008	2009	2010
Sales (in Rs. 000)	26	26	44	42	108	120	166

33. Find the points of local maxima and minima of the function $f(x) = 3x^4 - 4x^3 + 5$ in $[-1, 2]$. Also find absolute maximum and minimum values.

34. a) It is 7:00 PM currently. What time (in AM or PM) will be in next 1500 hours?

b) A boat covers 32km upstream and 36km downstream in 7 hours. Also, it covers 40km upstream and 48km, downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

35. Express $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrices and verify your result

Section - E (Case Studies)

36. The probability distribution of a random variable X is given as under:

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases} \text{ where } k \text{ is a constant}$$

Based on above information

- Find k
- Find $P(X < 4)$
- Find $E(X)$ or Find $E(3X^2)$

37. Two schools P and Q decided to award their selected students for the values of discipline and honesty in the form of prizes at the rate of ₹ x and ₹ y respectively. School P decided to award respectively 3, 2 students a total prize money of ₹2300 and school Q decided to award respectively 5, 3 students a total prize money of ₹3700.

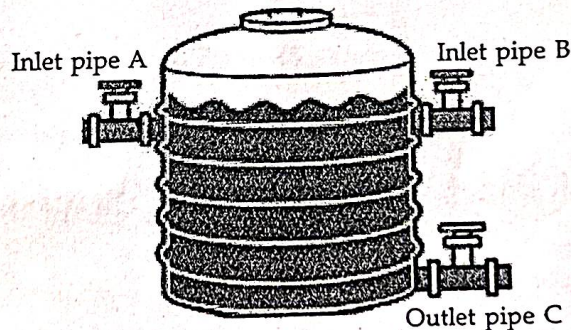
Based on the above information, answer the following questions:

- Write the matrix equation representing the above situation.
- Find the value of the determinant of coefficients of x and y .
- Find the values of x and y (use Cramer's rule)

OR

Find the inverse of matrix A.

38. An overhead water tank has three pipes A, B and C attached to it (as shown in figure given below). The inlet pipes A and B can fill the empty tank independently in 15 hours and 12 hours respectively. The outlet pipe C alone can empty a full tank in 20 hours.



Based on the above information, answer the following questions. Show steps to support your answers.

- For a routine cleaning of the tank, the tank needs to be emptied. If pipes A and B are closed at the time when the tank is filled to two-fifth of its total capacity, how long will pipe C take to empty the tank completely?
- How long will it take for the empty tank to fill completely, if all the three pipes are opened simultaneously?
- On a given day pipes A, B and C are opened (in order) at 5 a.m., 8 a.m. and 9 a.m. respectively, to fill the empty tank. In how many hours will the tank be filled completely?

OR

Given that the tank is half-full, only pipe C is opened at 6 a.m., to empty the tank. After closing the pipe C and an hour's cleaning time, tank is filled completely by pipes A and B together. What is the total time taken in the whole process?